The objectives for this presentation are:

- Provide a non-technical introduction to the logic behind covariance based structural equation modelling (SEM)
- Introduce the casual diagramming approach and concepts underlying it
- Contrast SEM to other methods (in particular multiple regression) and demonstrate why accounting for measurement error using SEM is very important
- Show how one can obtain more information regarding the generalizability of an instrument across various cultures
- Assess the relative importance of factors in a theoretical model for different cultures (i.e., the contingent effect of culture)
• SEM with causal diagrams involve three primary components:
  – indicators (often called manifest variables or observed measures/variables)
  – latent variable (or construct, concept, factor)
  – path relationships (correlational, one-way paths, or two way paths).

Indicators are normally represented as squares. For questionnaire based research, each indicator would represent a particular question.

Latent variables are normally drawn as circles. In the case of error terms, for simplicity, the circle is left off. Latent variables are used to represent phenomena that cannot be measured directly. Examples would be beliefs, intention, motivation.
Correlation between two variables. We assume that the indicator is a perfect measure for the construct of interest.

Multiple regression with two independent variables
\[ y = b_1X_1 + b_2X_2 + \text{error} \]
Impact of Measurement error on correlation coefficients

Correlated Two Factor Model
correlation matrix of indicators

\[
\begin{array}{|c|c|c|c|}
\hline
X_{11} & X_{12} & X_{21} & X_{22} \\
\hline
1.000 & & & \\
0.810 & 1.000 & & \\
0.576 & 0.576 & 1.000 & \\
0.576 & 0.675 & 0.640 & 1.000 \\
\hline
\end{array}
\]

Fit Function = \(\ln|\sum + \frac{S}{\sum} - \ln|\Sigma| - p\)

sample set for cultural group 1

sample set for cultural group 2
/TITLE
Testing For the Invariance of a Two Factor Model
Test For Equality of Factor Loadings
Group1=North America

/SPECIFICATIONS
CAS=318; VAR=6; ME=ML;
/EQUATIONS
V1 = *F1 + E1;
V2 = *F1 + E2;
V3 = *F1 + E3;
V4 = *F2 + E4;
V5 = *F2 + E5;
V6 = *F2 + E6;
/MATRIX
1.000
.929 .952 .834 .741 .712
1.000 .975 .975 .975 .975 .975
.975 .975 .975 .975 .975 .975
.834 .834 .834 .834 .834 .834
.834 .834 .834 .834 .834 .834
.959 .959 .959 .959 .959 .959
/VARIANCES
F1 TO F2 = 1; E1 TO E6 = *;
/COVARIANCES
F1 TO F2 = *;
/CONSTRAINTS
(1,V1,F1)=(1,V2,F1)
(1,V2,F1)=(2,V2,F1)
(1,V3,F1)=(2,V3,F1)
(1,V4,F2)=(2,V4,F2)
(1,V5,F2)=(2,V5,F2)
(1,V6,F2)=(2,V6,F2)
/LMTEST
/END

/TITLE
Testing For the Invariance of a Two Factor Model
Test For Equality of Factor Loadings
Group2=Hong Kong

/SPECIFICATIONS
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/EQUATIONS
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V2 = *F1 + E2;
V3 = *F1 + E3;
V4 = *F2 + E4;
V5 = *F2 + E5;
V6 = *F2 + E6;
/MATRIX
1.000
.975 .975 .746 .748 .731
.975 .975 .975 .975 .975
1.000 .999 .976 .999 .999
.767 .767 .766 .767 .767
.767 .767 .766 .767 .767
.9541 .9541 .9541 .9541 .9541
/VARIANCES
F1 TO F2 = 1; E1 TO E6 = *;
/COVARIANCES
F1 TO F2 = *;
/CONSTRAINTS
(1,V1,F1)=(2,V1,F1)
(1,V2,F1)=(2,V2,F1)
(1,V3,F1)=(2,V3,F1)
(1,V4,F2)=(2,V4,F2)
(1,V5,F2)=(2,V5,F2)
(1,V6,F2)=(2,V6,F2)
/END
STATISTICS FOR MULTIPLE POPULATION ANALYSIS

ALL EQUALITY CONSTRAINTS WERE CORRECTLY IMPOSED

GOODNESS OF FIT SUMMARY

INDEPENDENCE MODEL CHI-SQUARE = 6575.377 ON 30 DEGREES OF FREEDOM

INDEPENDENCE AIC = 6515.37744  INDEPENDENCE CAIC = 6356.85342

MODEL AIC = 191.51708  MODEL CAIC = 75.26613

CHI-SQUARE = 235.517 BASED ON 22 DEGREES OF FREEDOM

PROBABILITY VALUE FOR THE CHI-SQUARE STATISTIC IS LESS THAN 0.001

BENTLER-BONETT NORMED FIT INDEX= 0.964
BENTLER-BONETT NONNORMED FIT INDEX = 0.956
COMPARATIVE FIT INDEX = 0.967

LAGRANGE MULTIPLIER TEST (FOR RELEASING CONSTRAINTS)

CONSTRAINTS TO BE RELEASED ARE:

CONSTR: 1   (1,V1,F1)-(2,V1,F1)=0;
CONSTR: 2   (1,V2,F1)-(2,V2,F1)=0;
CONSTR: 3   (1,V3,F1)-(2,V3,F1)=0;
CONSTR: 4   (1,V4,F2)-(2,V4,F2)=0;
CONSTR: 5   (1,V5,F2)-(2,V5,F2)=0;
CONSTR: 6   (1,V6,F2)-(2,V6,F2)=0;

UNIVARIATE TEST STATISTICS:

<table>
<thead>
<tr>
<th>NO</th>
<th>CONSTRAINT</th>
<th>CHI-SQUARE</th>
<th>PROBABILITY</th>
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<tbody>
<tr>
<td>1</td>
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<td>0.732</td>
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<td>6</td>
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<td>0.324</td>
<td>0.569</td>
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<td>CHI-SQUARE</td>
<td>D.F.</td>
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<td>2</td>
</tr>
<tr>
<td>3</td>
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<td>CONSTR: 6</td>
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<td>5</td>
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<td>6</td>
<td>CONSTR: 1</td>
<td>26.146</td>
<td>6</td>
</tr>
</tbody>
</table>

- If the loadings were all invariant, the next constraint typically is to fix the correlation between the factors to be equal.
- If that yields a good fit - test of equal error terms would be next.
- Might consider testing the latent mean structures.
- Should note that prior to even doing a multiple-group test, the test of the model for each sample should be done.
• The logic just applied for the two factor model can similarly be applied in a causal model.
• In this situation, you can test to see if the paths are equivalent between groups.
• But prior to doing this, you’d wish to make sure the measurement model (indicators to factors) are appropriate.

• Relative importance of factors across cultures

Results from country 1

Results from country 2
• If the results were obtained by multiple regression, our conclusions of differences may be incorrect due to measurement error.
• Examine whether the reporting errors vary substantially between groups
• Unless we account for differences in the error term, the group with larger error terms will result in smaller least square estimates of the paths
• If certain paths are smaller due to measurement error, the remaining paths may be overinflated

• Example (Wolfle, 1987) looked at the differences between reports of parental status by parents and their high school children.
• Groups separated according to whites and blacks
• Three factor model consisting of:
  – father’s occupation
  – father’s education
  – mother’s education
A series of multiple group analyses resulted in the following:

- covariance of error between reports of education for mother and father estimated
- factor loading patterns were similar (invariance constraint was accepted)
- the true score variances and covariance were equal for whites and blacks
- error variances for black and white parents were equal
- thus both black and white parents report their status with equal reliability
• But the errors with which black high school seniors report their parents’ socioeconomic characteristics were consistently larger than those of white high school seniors.

• Except for white children reporting their father’s occupation, children in general reported their parents’ socioeconomic traits with greater error.

• Unless these differences in the error terms are accounted for, the result may lead to the erroneous conclusion that socioeconomic factors have less influence for blacks than whites on children performance or other dependent variables.